Calculus and the Projectile Launcher - The Ring of Fire!

I. Basic Equations

From algebra and trigonometry, when a ball is thrown (or launched) at an angle θ from an initial position of s_0 ft th an initial velocity of v_0 ft/sec, the motion of the ball can be modeled by the parametric equations:

$$x(t)=(v_0\cos\theta)t \quad \text{and} \quad y(t)=-16t^2+(v_0\sin\theta)t+s_0$$

Your group's angle θ will be: ________ degrees.

II. Collecting the Data

Using the projectile launcher, we need to measure two pieces of data. (Everything else can be calculated!)

1. The initial position/height s_0 of the ball when it is launched.

   s_0 = ____________________ in = __________________ ft

2. The horizontal distance **d** that the ball travels when it is launched horizontally ($\theta = 0°$) off the table.

   d = ____________________ in = ____________________ ft

III. Calculating the Initial Velocity

The first major value we need to calculate is the initial velocity (v_0) of the ball when it is launched. To calculate this we first need to find how long the ball was in the air when it is launched from an angle of 0°.

bstitute a 0° angle and the value s_0 that we measured into the parametric equations and then simplify both equations.

x = ______________________________ = ______________________________

y = ______________________________ = ______________________________

Knowing that the height (vertical position) of the ball is zero when it hits the ground, substitute **0** for **y** in your "y-equation" above and then solve it for **t**. (The variable **t** should be your only unknown in that equation!) Show your work below.

We don't want to "round-off" too much, so record the time it takes for the ball to hit the ground accurate to 3 decimal places.

t = ______________________ sec

Now substitute this value for **t** and the value for **d** that was previously measured into your "x-equation" and solve for v_0.

v_0 = ________________ ft/sec

IV. A Model for the Ball in Flight

To summarize so far, the ball was launched at a height of ____________ ft , with an initial velocity of __________ ft/sec. And my group is to use an angle of ____________ degrees.

Write the **parametric equations** that will model the flight of the ball when launched at this angle. Evaluate the cosine and sine of your angle θ and simplify each equation, keeping three decimal places for each number.

x(t) = ______________________________ = ______________________________

y(t) = ______________________________ = ______________________________

V. Calculating the Maximum Height and Horizontal Distance the Ball Will Travel

We are going to find the maximum height and the horizontal distance that the ball traveled both **analytically** and **graphically**. Then, of course, we will check these results by using our projectile launcher!

1. Analytically with Calculus

From our study of calculus applied to parametric equations, we know that a curve defined parametrically will have a horizontal tangent line when **dy/dx=0**. We also know that given the parametric equations **x=f(t)** and **y=g(t)**, we find **dy/dx** by the "short" formula:

dy/dx = ______________________

Calculate **dy/dt** (or f'(t)) and **dx/dt** (or g'(t)) for your parametric equations.

dy/dt = ______________________________

dx/dt = ______________________________

Since **dy/dx** is a ratio, it will equal zero when ______________________________________

Set this expression equal to zero and solve it for **t**.

dy/dt=0 when **t=**__________

Evaluate **y(t)** at this **t** value to find the **maximum height** of the ball. **y(** _______ **)** = __________

Evaluate **x(t)** at this time to find the **horizontal distance** from the launcher when it reaches the maximum height? **x(** ________ **)** = __________

To find the **horizontal distance that the ball travels** before it hits the ground, we need to first find the **time** it takes for the ball to hit the ground. To do this, set **y(t)= 0** in your "y-equation" above. This gives you the **quadratic equation**:

0 = ________ t^2 + ________________ **t** + ________________.

This equation can be solved using the **Quadratic Formula** with:

a = ____________ , **b =** ____________________, and **c =** ______________________

Put these values into the **Quadratic Formula** and then solve for t. Show your work below.

Results from the **Quadratic Formula**: **t =** ____________ and **t =** ____________

Only one of these values of **t** makes sense to the problem. Why?

____

Substitute the "good value" for **t** into your "x-equation" to find the horizontal distance that the ball will travel.

x = ____________________ ft.

Now, let's see if we confirm all of these values **graphically**.

2. Graphically
On your calculator, graph the parametric equations that model the flight of the ball.

TRACE on your graph to determine the **maximum height** of the ball and the **time** that the ball reaches this height. Also take a reading of the **horizontal distance** when the ball reaches its maximum height. While tracing, you may need to enter values for **t** to get accurate maximum height, horizontal distance, and time readings.

Complete the statement: At an angle of _________ degrees, the ball reaches a maximum height of ___________ ft, which is ___________ inches. It occurs at a time of __________ secs, and at a distance ___________ ft, which is ___________ inches, horizontally from the launcher.

How do these values compare to the values you found analytically? ____________________________

Now trace to find the **horizontal distance** that the ball travels when it hits the ground, and the **time** it takes to travel this distance. (Hint: You need to find the value of **t** and the **x-value** that makes the **y-value** very close to zero.)

How does this compare to your answer above?

Complete the statement: At an angle of _________ degrees, the ball hits the ground __________ ft, which is ____________ inches, horizontally from the launcher after ___________ secs.